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Methodology Article

Techniques for Computing the Probability of a Gambler's Ruin with Applications to Playing Roulette

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Abstract: An interest in gambling has greatly increased over the last few decades with the more common use of slot machines and online gambling, especially sports betting. A concern that has been publically raised is addiction and eventual gambler's ruin (loss of all money). In this paper we provide a solution to the Gambler's Ruin problem in regards to roulette. We compute the probability of a gamblers ruin with applications to the various betting opportunities playing roulette by determining the W+1 roots of the relevant polynomials and from there determine the probability of a gamblers ruin. We find situations where the payoff becomes higher, the probability of ruin becomes lower. Lower goals of gain are associated with a lower probability of ruin and larger bets and larger odds payoff also increase the probability of ruin.

Keywords: Odds, Probability, Gamblers Ruin, Roulette

1. Introduction

Gambling has a large past in human history along with a variety of associated problems, including addictions and financial ruin [1-7]. Much published on gamblers ruin assume the player is playing a game where on each individual play the player either wins \$1 (with probability q) and loses \$1 with (probability p) [8-12]. Many casinos provide opportunity to play games where the return on a win (\$W) can considerably greater than even money (\$W =\$1). This case is handled in the publication by Katriel [13]. He considers the case when the player plays the game and only stops when the players fortune goes to zero. Then if the game is in favor of the casino (the expected return to the player on each play is negative) the probability that your fortune will eventually go to zero (lose all of your fortune is equal to 1 [13]. On the other hand, if the player applies a strategy where he may stop play if his fortune reaches a target amount above his starting fortune. In that case the probability of ruin is not necessarily equal to 1. We will consider the case when a person will start with the fortune of M dollars and will play until the player either loses all of the M dollars or raises their fortune to A dollars. This case has been considered by H. L. Skala [14].

2. Solution to the Gambler's Ruin Problem

We assume that the player starts with M points (dollars) and plays a game where he either loses 1 point with probability p, or wins W points with probability q=1–p. We will let P(M) denote the probability of a gambler's ruin (loses his entire fortune) when he starts with an initial fortune of M.

Note that P(0)=1. If the player also stops when their fortune exceeds \$A, then

$$P(A)=0,\ P(A+1)=0,\dots,\ P(A+W-1)=0.$$

This is different than the side conditions assumed by Skala (2). He assumes that the player is playing against an adversary who begins with a beginning fortune of \$(A-M) and the game stops when the adversary's fortune is less than \$W (hence cannot cover the bet). This would be the case if the player's fortune has reached \$(A-W), \$ (A-W+1), \$(A-W+2),…\$A. In this case:

P(A-W)=0, P(A-W+1)=0,…, P(A)=0.

Note that if the player currently has a fortune of \$X and can win \$W with probability q and lose \$1 with probability p, then

$$P(X) = pP(X-1) + qP(X+W)$$

and $P(0) = 1. P(A) = 0, P(A+1) = 0, \cdots, P(A+W-1) = 0$

The formula for $P(X)$ can be found by finding the W+1 roots of the polynomial

$$qz^{W+1} - z + p = 0$$

$$r_1, r_2, \cdots, r_{W+1}$$

Roots of polynomials can be found using programs such as Mathematica or Matlab. Once the roots are found, then the formula is $P(X) = c_1 r_1^X + c_2 r_2^X + \cdots + c_{L+1} r_{L+1}^X$

The conditions $P(0) = 1. P(A) = 0, P(A+1) = 0, \cdots, P(A+W-1) = 0$ are satisfied if:

$$c_1 + c_2 + \cdots + c_{L+1} = 1$$

$$c_1 r_1^A + c_2 r_2^A + \cdots + c_{L+1} r_{L+1}^A = 0$$

$$c_1 r_1^{A+1} + c_2 r_2^{A+1} + \cdots + c_{L+1} r_{L+1}^{A+1} = 0$$

$$\vdots$$

$$c_1 r_1^{A+W-1} + c_2 r_2^{A+W-1} + \cdots + c_{L+1} r_{L+1}^{A+W-1} = 0$$

or

$$\begin{bmatrix} 1 & 1 & \cdots & 1 & 1 \\ r_1^A & r_2^A & \cdots & r_L^A & r_{L+1}^A \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ r_1^{A+W-2} & r_2^{A+W-2} & \cdots & r_L^{A+W-2} & r_{L+1}^{A+W-2} \\ r_1^{A+W-1} & r_2^{A+W-1} & \cdots & r_L^{A+W-1} & r_{L+1}^{A+W-1} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_L \\ c_{L+1} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

orinmatrixform $A\vec{c} = \vec{b}$where

$$A - \begin{bmatrix} 1 & 1 & \cdots & 1 & 1 \\ r_1^A & r_2^A & \cdots & r_L^A & r_{L+1}^A \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ r_1^{A+W-2} & r_2^{A+W-2} & \cdots & r_L^{A+W-2} & r_{L+1}^{A+W-2} \\ r_1^{A+W-1} & r_2^{A+W-1} & \cdots & r_L^{A+W-1} & r_{L+1}^{A+W-1} \end{bmatrix}, \vec{c} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_L \\ c_{L+1} \end{bmatrix} \text{and} \vec{b} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

Note: the elements of A may be complex-numbers (calculated from the roots of a polynomial). Thus one can also consider the elements of $\vec{c}$ and $\vec{b}$ to be complex numbers.

Thus $A = A_1 + iA_2, \vec{c} = \vec{c}_1 + i\vec{c}_2$ and $\vec{b} = \vec{b}_1 + i\vec{b}_2$ where $A_1, \vec{c}_1$ and $\vec{b}_1$ are there all elements of $A, \vec{c}$ and $\vec{b}$ and $A_2, \vec{c}_2$ and$\vec{b}_2$ are the imaginary elements of $A, \vec{c}$ and$\vec{b}$

Then the equation $A\vec{c} = \vec{b}$can be written$(A_1 + iA_2)(\vec{c}_1 + i\vec{c}_2) = \vec{b}_1 + i\vec{b}_2$

Or $(A_1\vec{c}_1 - A_2\vec{c}_2) + i(A_2\vec{c}_1 + A_1\vec{c}_2) = \vec{b}_1 + i\vec{b}_2$

Thus $A_1\vec{c}_1 - A_2\vec{c}_2 = \vec{b}_1$ and$A_2\vec{c}_1 + A_1\vec{c}_2 = \vec{b}_2$

And $\begin{bmatrix} A_1 & -A_2 \\ A_2 & A_1 \end{bmatrix} \begin{bmatrix} \vec{c}_1 \\ \vec{c}_2 \end{bmatrix} = \begin{bmatrix} \vec{b}_1 \\ \vec{b}_2 \end{bmatrix}$ which has solution $\begin{bmatrix} \vec{c}_1 \\ \vec{c}_2 \end{bmatrix} = \begin{bmatrix} A_1 & -A_2 \\ A_2 & A_1 \end{bmatrix}^{-1} \begin{bmatrix} \vec{b}_1 \\ \vec{b}_2 \end{bmatrix}and\vec{c} = \vec{c}_1 + i\vec{c}_2$

These solutions can be obtained using Excel: MULT (MINVERSE(A*), b*).

Where $A^* = \begin{bmatrix} A_1 & -A_2 \\ A_2 & A_1 \end{bmatrix}$ and $b^* = \begin{bmatrix} \vec{b}_1 \\ \vec{b}_2 \end{bmatrix}$

Once $\vec{c} = \vec{c}_1 + i\vec{c}_2$ has been found then the probability of ruin b beginning with an initial fortune of \$$M$ is:

$$P(M) = c_1 r_1^M + c_2 r_2^M + \cdots + c_{L+1} r_{L+1}^M$$

Computing the elements of A, A_1 and A_2,

Many of the elements the matrix A are of the form r_k^n where$r_k = x_k + y_x i$

One can then put$r_k$into the polar form $r_k = x_k + y_x i = R_k e^{i\theta_k} = R_k[cos(\theta_k) + isin(\theta)]$

Where $R_k = \sqrt{x_x^2 + y_x^2}and\frac{y_k}{x_k} = tan(\theta_k) or \theta_k = tan^{-1}\left(\frac{y_k}{x_k}\right) = atan\left(\frac{y_k}{x_k}\right)$

Then $r_k^n = R_k^n e^{in\theta_k} = R_k^n[cos(n\theta_k) + isin(n\theta_k)]$ (elementsof$A$).

And $Re(r_k^n) = R_k^n[cos(n\theta_k)]$(elementsof$A_1$).

And $Im(r_k^n) = R_k^n[sin(n\theta_k)]$(elementsof$A_2$).

3. Application to Roulette

Roulette is a game that is available for play in both casinos and online casinos. A roulette wheel has 38 pockets (American wheel), 37 pockets (for the European wheel).

Figure 1. *American and European Roulette wheels.*

The pockets are numbered 1 to 36,0,00 for the American wheel (1 to 36, 0 for the European wheel).

Many different types of bets can be made with varying pay offs and odds of winning.

These are made on a Roulette table illustrated below:

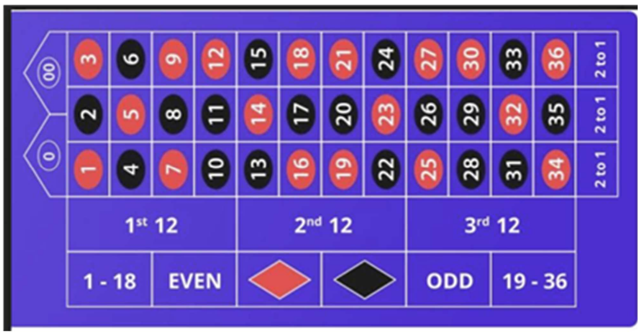

Figure 2. *Roulette Table.*

3.1. Different Betting Possibilities

Table 1. *The different betting possibilities together with L (cost of playing the game), W (amount won in the event of a win), q (the probability of winning W), q (the probability of losing 1 unit).*

Bet type	W	L	q	p
Inside Bets				
Single Number	35	1	1/38	37/38
Split Bets	17	1	2/38	36/38
Street Bets:	11	1	3/38	35/38
Corner Bets or Square Bets	8	1	4/38	34/38
Six Line Bet	5	1	6/38	32/38
Top Line Bet:	6	1	5/38	33/38
Outside Bets				
Column Bets	2	1	12/38	26/38
Dozens Bets	2	1	12/38	26/38
Odd/Even Bets	1	1	18/38	20/38
Red/Black Bets	1	1	18/38	20/38
1-18/19-36 (Low/High) Bets	1	1	18/38	20/38

The following is a list of the different betting possibilities. These are found on [15].

Inside Bets

Single Number Bets (also known as Straight-up Bets or Classic Bets): Players may wager on any individual number by placing chips on top of that number. If the number is hit on the next spin, this wager pays 35-1.

Split Bets: This bet is made on any two numbers that are adjacent on the roulette board and can be made by placing chips on the line between those two numbers. If either number is the winner on the next spin, this bet pays 17-1.

Street Bets: This bet is made on any row of three numbers on the roulette table and can be made by placing your chips on the edge of that row. If any of those three numbers win, the bet pays 11-1.

Corner Bets or Square Bets: This bet is made on a square off our numbers on the roulette board and can be made by placing chips on the points shared by those four numbers. If any of the four numbers win, this bet pays 8-1.

Six Line Bet: This bet is made on two adjacent lines of three numbers each, for a total of six numbers. This bet is made by placing chips at the intersection between the two lines along the side of the board. If any of those six numbers win, the bet pays 5-1.

Top Line Bet: One additional bet is available to players in American roulette. This bet can be made on the five numbers 0-00-1-2-3. This bet pays 6-1; however, it is the only bet with a higher than normal house edge (7.9%), and should probably be avoided during play.

Outside bets: Outside bets are less risky than inside bets, but also come with lower pay outs. These wagers can be found on the "outside" of the roulette board, around the area where the numbers are listed. Outside bets available to players are as follows:

Column Bets: This bet covers one of the three columns of 12 numbers found on the roulette wheel. If any number in that column wins, the bet pays 2-1.

Dozens Bets: These bets cover one of three groups of 12 numbers (1-12, 13-24, and 25-36). If any number in that range wins, the bet pays 2-1.

Odd/Even Bets: These two bets cover all odd numbers or even numbers respectively. If any number of that type wins, this bet will pay even money.

Red/Black Bets: These two bets cover all numbers that have pockets of the appropriate color on the roulette wheel. If any number of that color wins, the bet pays even money.

1-18/19-36: These two bets cover all numbers in the ranges suggested by their names. If any number in that range should win, the bet pays even money.

3.2. Computation of Probability of a Gambler's Ruin

To compute the probability of a gambler's ruin we need to find the W+1 roots of the polynomial (using either the program Mathematica or Matlab).

$$qz^{W+1} - z + p = 0$$

$$r_1, r_2, \cdots, r_{W+1}$$

where$r_k = x_k + y_x i = R_k e^{i\theta_k} = R_k[cos(\theta_k) + isin(\theta)]$ and $R_k = \sqrt{x_x^2 + y_x^2}$and $\frac{y_k}{x_k} = tan(\theta_k)or\theta_k = tan^{-1}\left(\frac{y_k}{x_k}\right) = atan\left(\frac{y_k}{x_k}\right)$

Then we need to solve the equations (this can be done using Excel).

$\begin{bmatrix} A_1 & -A_2 \\ A_2 & A_1 \end{bmatrix}\begin{bmatrix} \vec{c}_1 \\ \vec{c}_2 \end{bmatrix} = \begin{bmatrix} \vec{b}_1 \\ \vec{b}_2 \end{bmatrix}$ which has solution $\begin{bmatrix} \vec{c}_1 \\ \vec{c}_2 \end{bmatrix} = \begin{bmatrix} A_1 & -A_2 \\ A_2 & A_1 \end{bmatrix}^{-1}\begin{bmatrix} \vec{b}_1 \\ \vec{b}_2 \end{bmatrix}$ and $\vec{c} = \vec{c}_1 + i\vec{c}_2$

where the elements of A_1 are

$$a_{ij}^{(1)} = \begin{cases} 1 & i = 1 \\ Re\left(r_j^{A+i-2}\right) & i = 2, 3, \cdots, W+1 \end{cases} = \begin{cases} 1 & i = 1 \\ R_j^{A+i-2}\left[cos\left((A+i-2)\theta_j\right)\right] & i = 2, 3, \cdots, W+1 \end{cases}$$

and the elements of A_2 are:

$$a_{ij}^{(2)} = \begin{cases} 0 & i = 1 \\ Im\left(r_j^{A+i-2}\right) & i = 2,3,\cdots,W+1 \end{cases} = \begin{cases} 0 & i = 1 \\ R_j^{A+i-2}\left[sin\left((A+i-2)\theta_j\right)\right] & i = 2,3,\cdots,W+1 \end{cases}$$

The elements of $\vec{b}_1$ are

$$b_i^{(1)} = \begin{cases} 1 & i = 1 \\ 0 & i = 2,3,\cdots,W+1 \end{cases}$$

The elements of $\vec{b}_2$ are

$$b_i^{(2)} = 0 \quad i = 1,2,\cdots,W+1$$

Then once$\vec{c}_1, \vec{c}_2$ and $\vec{c} = \vec{c}_1 + i\vec{c}_2$ have been determined then the probability of a gambler's ruin beginning with an initial fortune of M is

$$P(M) = c_1 r_1^M + c_2 r_2^M + \cdots + c_{L+1} r_{L+1}^M$$

This has been used to evaluate the following table.

Table 2. *Probability of a Gambler's Ruin A (the initial fortune) and M (the final desired fortune of the gambler).*

Initial	Final	W=1, L=1	W=2, L=1	W=5, L=1	W=8, L=1	W=11, L=1	W=17, L=1	W=35, L=1
M	A	Prob of Ruin	Prob of Ruin	Prob of Ruin	Prob of Ruin	Prob of Ruin	Probof Ruin	Prob of Ruin
10	11	14.6%	17.0%	4.2%	1.3%	0.3%	0.0%	0.0%
100	110	65.1%	42.4%	21.0%	13.9%	13.1%	10.5%	2.3%
1000	1100	100.0%	100.0%	88.2%	73.2%	61.9%	46.2%	24.5%
10	20	74.1%	64.0%	55.0%	48.7%	50.4%	48.5%	3.0%
100	200	100.0%	99.5%	89.4%	78.8%	72.3%	64.7%	57.2%
1000	2000	100.0%	100.0%	100.0%	100.0%	100.0%	99.8%	95.5%

4. Conclusion

An examination of Table 2 shows when your goal is a 10% increase in your fortune (10 to 11, 100 to 110, 1000 to 1100), the probability of ruin is much less compared to a goal of 100% increase in your fortune (10 to 20, 100 to 200, 1000 to 2000). Also, when your fortune is a larger number of units (1000 to 100 to 10), the probability of ruin is increased. The probability of ruin becomes large and close to 100% when you make larger bets. Also, examining the table 1 observes that the probability of ruin as the payoff odds goes up (W=1 to L=1 compared to W=35 to L=1).

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